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Toward a Consistent Plate Theory

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IN Ref. 1 a beam theory has been given, avoiding some of the traditional contradictions normally tolerated in the elementary- and Timoshenko-type shear deformation beam theories. In this theory a cubic variation of the direct stress and parabolic variation shear stress across the depth of the beam have been considered. Numerical results pertaining to a tip-loaded cantilevered beam indicate that the theory of Ref. 1 is capable of giving stress distributions corresponding to Timoshenko-type shear deformation theory in regions where transverse shear effect is significant, and elementary-theory-type stress distributions in regions where transverse shear effect is insignificant, with smooth transition patterns in between. In this Note, the corresponding theory for flexure of plates is presented.

Figure 1 shows a typical plate subjected to a dynamic load $f(x,y,t)$. Following Ref. 1, the displacement field is chosen in the form

$$w(x,y) = w_b(x,y) + w_s(x,y)$$
$$u(x,y,z) = -zw_{b,x} - p(z)\phi(x,y)$$
$$v(x,y,z) = -zw_{b,y} - p(z)\psi(x,y) \tag{1}$$

where w_b is the partial deflection due to bending and w_s the partial deflection due to shear. ϕ and ψ represent the nonclassical in-plane displacement distribution

$$p = z(1 - 4z^2/3h^2) \tag{2}$$

The strain-displacement relations are

$$\epsilon_x = \frac{\partial u}{\partial x}, \quad \epsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \quad \epsilon_y = \frac{\partial v}{\partial y}$$
$$\epsilon_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \quad \epsilon_z = 0, \quad \epsilon_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \tag{3}$$

and the stress-strain relations are

$$\sigma_x = \lambda(\epsilon_x + \mu\epsilon_y), \qquad \sigma_y = \lambda(\epsilon_y + \mu\epsilon_x)$$
$$\sigma_{ij} = \lambda g \epsilon_{ij}; \quad i \neq j, \quad i,j = x,y,z \tag{4}$$

where

$$\lambda = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \quad \text{and} \quad \mu = \frac{\nu}{1-\nu}$$

which correspond to $\epsilon_z = 0$ and $\sigma_z \neq 0$. However, if one assumes ϵ_z and $\sigma_z = 0$, as in the case of the classical thin-plate theory, the corresponding values will be $\lambda = E/(1-\nu^2)$ and $\mu = \nu$. In either case,

$$g = G/\lambda$$

where G is the shear modulus, E Young's modulus, and ν Poisson's ratio.

Utilizing Hamilton's principle, the equations of motion may be obtained as

$$\lambda[I\nabla^4 w_b + C\nabla^2(\phi_{,x} + \psi_{,y})] - \rho I \nabla^2 \ddot{w}_b + \rho C(\ddot{\phi}_{,x} + \ddot{\psi}_{,y}) + \rho h[\ddot{w}_b + \ddot{w}_s] = f(x,y,t)$$
$$\lambda g[h\nabla^2 w_s - c(\phi_{,x} + \psi_{,y}) - \rho h[\ddot{w}_s + \ddot{w}_b] = f(x,y,t)$$
$$\lambda[D(\phi_{,xx} + (\mu+g)\psi_{,xy}) + g\phi_{,yy} + C\nabla^2 w_{b,x} + gcw_{s,y} - gd\phi] - \rho[C\ddot{w}_{b,x} + D\ddot{\phi}] = 0$$
$$\lambda[D(\psi_{,yy} + (\mu+g)\phi_{,xy} + g\psi_{,xx}) + C\nabla^2 w_{b,y} + gcw_{s,x} - gd\psi] - \rho[C\ddot{w}_{b,y} + D\ddot{\psi}] = 0 \tag{5}$$

where the overdot indicates derivation with time and the $,x$ or $,y$ subscript indicates derivation with respect to the variable indicated and

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

$$I = h^3/12, \quad C = h^3/15, \quad D = 19h^3/315, \quad c = 2h/3, \quad d = 8h/15 \tag{6}$$

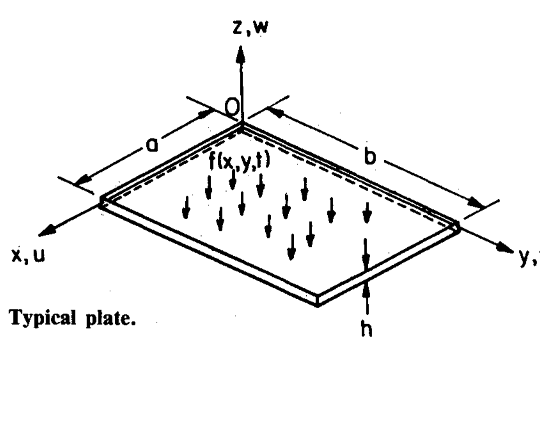

Fig. 1 Typical plate.

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Table 1 Frequency spectrum of simply supported rectangular plates ($E=6.9\times10^{10}$ N/m^2, $\nu=0.3$, $a/b=2$, $a=1$, $\rho=2720$ kg/m^3) $\lambda_{fp}=\omega^2\rho h a^2 b^2/\pi^4 \lambda I(a^2+b^2)$

	$a/h=25$			$a/h=50$		
Parameter	ET	SDT	PT	ET	SDT	PT
Bending mode						
λ_{fp}	1.0	0.980	0.977	1.0	0.9948	0.9941
w_b	1.0	1.0	1.0	1.0	1.00	1.0
w_s	—	0.014	0.0001	—	−0.004	0.9×10^{-5}
ϕ	—	—	−0.062	—	—	−0.017
ψ	—	—	−0.129	—	—	−0.033
Shearing mode						
λ_{fp}	—	10,946	12,621	—	172,477	199,077
w_b	—	−0.993	−0.991	—	−0.998	−0.998
w_s	—	1.00	1.0	—	1.0	1.0
ϕ	—	—	2.52	—	—	2.64
ψ	—	—	−3.90	—	—	−4.26
In-plane mode 1						
λ_{fp}	—	—	7984	—	—	126,767
w_b	—	—	−0.00187	—	—	−0.0005
w_s	—	—	0.00159	—	—	0.0004
ϕ	—	—	1.0	—	—	1.0
ψ	—	—	−0.10	—	—	−0.103
In-plane mode 2						
λ_{fp}	—	—	1812	—	—	28,427
w_b	—	—	−0.1097	—	—	−0.110
w_s	—	—	0.1099	—	—	0.110
ϕ	—	—	0.441	—	—	0.444
ψ	—	—	1.0	—	—	1.0

Boundary conditions:

Simply supported edge at $x=$const:

$$w_b=w_s=w_{b,xx}=\phi_{,x}=\psi=0 \tag{7}$$

Simply supported edge at $y=$const:

$$w_b=w_s=w_{b,yy}=\psi_{,y}=\phi=0 \tag{8}$$

The Mindlin-type shear deformation theory (SDT) can be obtained from Eq. (5) by setting $\phi=\psi=0$. The classical thin-plate theory, referred to herein as ET, can be obtained by setting $w_s=\phi=\psi=0$. Table 1 gives the frequency spectrum of rectangular plates for two values of a/h ratios. Results obtained by the present theory (PT) are compared with ET and SDT. λ_{fp} is the frequency parameter and w_b, w_s, ϕ, and ψ give amplitudes of bending and shear partial deflections, and nonclassical in-plane displacements. Modes with w_b very much larger than the other three are the classical bending modes. When w_b and w_s are of the same order of magnitude, it will be called the shearing mode. The other two modes involve primarily in-plane motion and will be called in-plane modes. All three theories—ET, SDT, and PT—predict bending modes. The accuracy of the estimate to λ_{fp} improves only slightly in the present theory as compared to SDT. However, it must be noted that w_s is considerably overestimated in SDT. This implies that in SDT, significant distortions in dynamic response are possible. Some results pertaining to dynamic response of beams are given in Ref. 2. The in-plane displacement components ϕ and ψ are larger than the shear partial deflection w_s, and are entirely missed in SDT and ET. The in-plane displacements in shearing modes are also much larger than the transverse deflections. The frequency of the shearing modes is higher than the frequencies of the in-plane modes. Thus the SDT misses the relatively more important in-plane modes, and introduces significant distortions in the mode shapes by overestimating the transverse shear partial deflection. Thus there is a need to consider higher-order theories.

The theory given in this Note is an improvement over the formulation of Ref. 3. The theory given in Ref. 3 corresponds to the $n=1$ case of Ref. 4 and has no provision for nonzero transverse shear strain at points in the plate where $u=v=w=0$, such as those on a fixed boundary, this limitation is overcome in the present model.

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Stationary Response to Second-Order Filtered White-Noise Excitation

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Introduction

FILTERED white noise, i.e., the stationary output of a stable linear system due to white-noise input, is an acceptable excitation model used in analyzing many physical problems.[1] In this Note the correlation functions of the response due to second-order filtered white-noise excitation are obtained through the time-domain modal analysis.[2,3] It is found that the stationary response per se is also filtered white noise. A resonance phenomenon similar to that in deterministic vibrations is noticed.

Excitation Model

The excitation considered here is second-order stationary filtered white noise $w(t)$ with zero mean and the following correlation function

$$R_w(\tau)=ge^{q|\tau|}+\bar{g}e^{\bar{q}|\tau|} \tag{1}$$

where q is the eigenvalue of the second-order linear filter,

$$q=-a+jb, \qquad a>0$$

and g is a complex number. With $g=b-ja$, the corresponding power spectral density of $w(t)$ is

$$G_w(\omega)=2a\left[\frac{2b+\omega}{a^2+(\omega+b)^2}+\frac{2b-\omega}{a^2+(\omega-b)^2}\right]$$

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